DOCUMENT RESUME

ED 048 942 PS 004 530

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TITLE Differential Performance of Kindergarten Children on Transitivity of Three Matching Relations.
INSTITUTION Georgia Univ., Athens.
PUB DATE 4 Feb 71
NOTE 26p.; Paper presented at the Annual Meeting of the American Educational Research Association, New York, New York, February 4, 1971

EDRS PRICE EDRS Price MF-$0.65 HC Not Available from EDRS.
DESCRIPTORS *Conservation (Concept), *Kindergarten Children, *Mathematical Concepts, Stimuli, Tables (Data), *Test Construction
IDENTIFIERS Equivalence, Order Relations, Transitive Property

ABSTRACT

The major objective of this study was to investigate differential performances among 5-year-old children when using transitivity of matching relations. Instruments were constructed to measure the subjects' (1) knowledge of matching relations, (2) ability to conserve the relations, and (3) proficiency in making inferences using the transitive property of the relations. Three specially designed tests, administered individually to 21 boys and 21 girls, assessed knowledge of relational terminology, level of conservation and performance on manipulative tasks (matching, comparing, judging quantitative relations among objects). A surprising result of this investigation was that subjects in the high category of conservation did not perform significantly better on transitivity than subjects in the low category of conservation for each relation. In general, differential performance on transitivity between equivalence and order relational groups occurred only within the high conservation level. An extensive comparison is made between present results and Smedslund's (1963a) data for conservation and transitivity of discontinuous quantity. (WY)

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Differential Performance of Kindergarten Children On Transitivity of Three Matching Relations

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PS 004530

One purpose of this study was to investigate differential performances among categories of five-year-old children when using transitivity of matching relations, where the categories were defined by an ability to conserve the relations involved. A second purpose was to ascertain whether an ability to make transitive inferences in the context of matching relations was relational specific. A third purpose was to ascertain whether an ability to use the transitive property of matching relations was influenced by stimulus conditions.

In mathematics, equivalence and order relations are inextricably involved in cardinal number. Even though Piaget (1964, p. 36) disagrees with Russell's definition of number, both men see equivalence or order relations involved in number. Russell (1956) has stated that "the number of a class is the class of all those classes that are similar* to it [p. 542]," while Piaget (1964) views number as "a synthesis of class inclusion and serial ordering . . . an indisociable synthesis [p. 39]." It has been argued that different conceptions of number lead to different instruction for children and different interpretation and different conduction of research (Van Engen, 1970). At present, however, in anyone's mathematical curricula for young children, activities based on one-to-one correspondence are presented, a practice grounded in mathematical structure. Little evidence has been presented by mathematics educators that such practices develop anything beyond an ability in a child to make overt comparisons between physical collections of objects. That is not to say that such evidence cannot be presented. In any such presentation, however, methodological issues which surround criteria for presence of concrete transitivity must be considered.

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*Similar, in this definition, means equivalent.

Paper presented at the Annual Meeting of the American Educational Research Association, New York, New York, February 4, 1971.

Clearly, one can classify a child as not being able to use transitivity when in fact he can, or classify a child as being able to use transitivity when in fact he cannot. In the first case, Smedslund (1963b) has discussed three possibilities; no understanding of instruction, failure to perceive the two initial comparisons, and forgetting. In the second case (Smedslund, 1963b) a child could obtain a correct response by guessing, by a perceptual discrimination, or by employing non-transitive hypotheses. Using these methodological credos, Smedslund constructed a test of concrete transitivity of length in which he utilized only one length relation "longer than" and one basic stimulus condition involving the Müeller-Lyer illusion. Certainly ascertainment of a symptom of transitivity in the case of one relation and one stimulus condition is encouraging, but for school mathematics, highly insufficient. For transitivity to be operative within a category of relations (e.g., length, matching, etc.) it must be operative across relations within that category across stimuli. The six methodological credos isolated by Smedslund do apply regardless of the relation or stimulus condition. In this study, to protect against classifying a child as not being able to use transitivity when in fact he can because of not understanding the instructions and failure to perceive the two initial comparisons, the relations "as many as," "more than." and "fewer than" were operationally defined. In the following operational definition, the sets are considered to be finite sets of physical objects.

A child is said to be able to establish a matching relation between set A and set B if, and only if, he forms pairs of elements, where one member of each pair is chosen from set A and the other is chosen from set B, until one or both of the sets are exhausted, and then deduces that there are as many a's as b's whenever the two sets are exhausted; or that there are fewer a's than b's whenever set A is exhausted and set B is not (also, in this case, he

deduces that there are more b's than a's).

A set of learning activities and a measuring instrument were designed in accordance with the above operational definition. The measuring instrument was administered at the end of the presentation of the learning activities to ascertain whether children could establish the relations and discriminate among them.

In order to protect against classifying a child as not being able to use the transitive property when in fact he can because of forgetting, the children were required to establish each of the two relations in the transitive paradigm before making the transitive inference. The role of forgetting in transitivity has been studied (Smedslund, 1960); the results indicate that it is not a factor.

In order to protect against a child's obtaining a correct response by guessing, three questions were asked of each child after he made the two initial comparisons. Two of these had a correct response of "no" and one had a correct response of "yes." The questions were: "Are there as many a's as b's?" "Are there fewer a's than b's?" "Are there more a's than b's?" Moreover, a criterion level which was based on a random model was set. Smedslund (1963b) controlled for guessing by introducing perceptual illusion. In this study, however, the more stringent criteria mentioned above were employed. A child not only had to use transitivity to deduce that, for example; A ˜ C in the case of the two hypotheses A ˜ B and B ˜ C, but also had to judge that there were not fewer A than B nor more A than B. It must be pointed out that no attempts were made to elicit verbal reasons from the children. Two reasons are advanced for this decision. First, it is clear that children need not give verbal explanations to make transitive inferences. Secondly, data presented by past researchers (Smedslund, 1963b) clearly show that a very close relationship holds between being able to offer adequate explanation and making correct judgements.

In order to protect against a child's making correct judgements based on perceptual discrimination and non-transitive hypotheses, only sets of objects with at least five members were used. Also the sets A and C differed by at most two objects.

It is desirable that a child, upon establishing a relation between sets A and B, and B and C, should be able to infer the relation between A and C regardless of the spacial arrangement or rearrangement of the objects, or whether or not the objects of A and C are actually visible during inference. Three stimulus conditions were thereby isolated for study; those spacial arrangements of A and C from which it was unlikely that cues would be obtained (neutral stimulus); those spacial arrangements of A and C from which was likely incorrect cues would be obtained (conflict stimulus); and a screening of the objects of A and C (screened stimulus).

It has been shown that children who do not conserve a particular relation have little chance of making a transitive inference using that relation (Carey and Steffe, 1968). For this reasen, the children of this study were classified into two conservation of matching relations groups, those meeting an established criterion and those not meeting the established criterion. It was expected that for those children who did not meet conservation criterion, the three stimulus conditions would have little effect since they would not be able to make a transitive inference. For those children who did meet conservation criterion, it was expected that the children would perform less well in the case of the conflict stimulus than on the other two stimulus conditions. The reasons for advancing this latter hypothesis are two-fold. First, Bruner (1966) found that five and six-year-old children markedly improved in conservation of liquid abilities under a screening of perceptual cues training session. In the present study the thinking was that the screened stimulus would force the child to focus on the only information available; i.e., the two hypotheses, and thereby eliminate solution by extraneous processes. Secondly, Braine (1959) has found an

occurrance of transitivity in subjects much younger than Smedslund or Piaget. According to the latter two, 50 percent of children achieve transitivity between seven and eight years of age, where the Braine data suggests this age to be about five. While Braine used nonverbal techniques, he also used stimuli which could be categorized under the rubric, "neutral stimulus."

Method

Tests

Instruments were constructed to measure the subjects (Ss) knowledge of the matching relations, ability to conserve the relations, and proficiency in making inferences using the transitive property of the relations. Evaluation was on a one-to-one basis in the presence of such concrete materials as checkers, tiles and colored wooden discs. The number of objects ranged from 5 to 10, and in the case of a comparison involving an order relation, the two sets differed in number by one.

Relation Test (RT). An 18-item test was constructed to determine if the children knew the relational concepts and terminology. Six items involved sets related by each of the relations "as many as," "more than," and "fewer than." In each item S was asked to match the sets A and B, where A and B were replaced by the name for the categories of objects (e.g., checkers and tiles). S was then asked three questions, and allowed time to respond to each as follows: (a) "Are there as many a's as b's?" (b) "Are there more a's than b's?" (c) "Are there fewer a's than b's?". The questions occurred in the above sequence if A and B were in fact equivalent, and in the sequence (b), (a), (c) or (c), (a), (b), if there were more a's than b's or fewer a's than b's, respectively. A child was said to establish the relation on an item provided he answered all three questions correctly.

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Conservation of Matching Relations Test (CMRT). Immediately after S had responded to the questions on an item of the RT, the set of objects denoted by A was physically transformed by the experimenter (E). E then repeated questions (a), (b), and (c) in a randomly-ordered sequence. If a child established the relation and responded correctly to all three questions which followed the transformation, he was said to conserve the relation of the item.

A diagram of an exemplary item involving conservation of the relation "as many as" is shown in Figure 1. The first diagram shows the item as the child

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Insert Figure 1 about here

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saw it initially. There were seven tiles in a column glued onto a piece of white cardboard which measured approximately 9 inches by 22 inches, and seven checkers nearby. E said "Match the checkers and the tile." After S had matched them as shown in (2), E asked the questions (a), (b) and (c) above, and allowed time for a response following each. E then spread the checkers as shown in (3) of Figure 1 and repeated the questions.

Transitivity of Matching Relations Test (TMRT). An 18-item transitivity test was constructed for each of the relations "as many as," "more than," and "fewer than." Each test was composed of six items involving each of the three stimulus conditions--neutral stimulus, screened stimulus, and conflict stimulus. Figure 2 shows a diagram of an item from the neutral stimulus subtest in which Ss were asked to use transitivity of "more than" in the presence of stimulus

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Insert Figure 2 about here

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objects arranged with no apparent perceptual bias. The child initially saw the eight checkers, seven tiles, and six stars arranged on a piece of white cardboard

measuring 14 inches by 22 inches as shown in (1). E instructed S to match the checkers and the tiles, as shown in (2). E then asked, "Are there more checkers than tiles?" After S's response he was instructed to match the tiles and the stars. When this was done, as indicated in (3), E asked "Are there more tiles than stars?" Following the response, E removed the tiles. While pointing to the appropriate collection, indicated in (4), Figure 2, E asked the following three questions: (a) "Are there as many checkers as stars?" (b) "Are there more checkers than stars?" (c) "Are there fewer checkers than stars?" These questions were presented in a randomly-ordered sequence and time was allowed for a response following each.

The same procedure was followed in the perceptual conflict situation, but the stimulus objects were arranged in columns. If the collections were not equivalent, the longer row contained fewer objects. If the sets were equivalent, the columns were different in length as well.

For a description of an item in the screening category, suppose for example, there were in fact fewer a's than b's and fewer b's than c's. The objects denoted by B and C were in columns of approximately equal lengths. The objects denoted by A were in a heap beside B when the child initially saw the items. E first instructed S to match A and B. When this was done, E asked "Are there fewer a's than b's?" After the child had agreed, E placed the objects denoted by A in a drinking cup which sat near A. E then instructed S to match B and C and asked "Are there fewer b's than c's?" When S had agreed, E placed the objects C in another cup. While pointing to the respective cups E then asked, "Are there as many a's as c's?" "Are there more a's than c's?" "Are there fewer a's than c's?" These questions were randomly sequenced and a response was encouraged following each question. A child was said to make a transitive inference on an item of TMRT provided the child agreed that the relation held between A and B and between B and C and provided that he responded correctly to all three concluding questions.

It should be noted that the items for the three stimulus categories within a relation contained the same number and kind of objects. The items of the three relational tests were analogous and differed only in the number of objects so that the respective relations held. Both CMRT and TMRT were balanced with respect to items proceeding from right to left and left to right.

Unit of Instruction

Seven 20-30 minute lessons were designed by the researchers to operationally define the relations for the children. Activities varied from children working individually or in pairs to those in which the group participated collectively. In the first case each child had his own set of concrete materials such as toys, buttons, checkers, tiles, and wooden discs. The group activities (e.g., flannel board activities) were characterized by children alternately manipulating the materials while the remaining children made observations. Dialogue or group discussions of the observations involved the relational terminology.

The first lesson dealt with establishing a language base by forming pairs of objects from two disjoint sets and calling the members of each pair "partners." In a later lesson the children were then given two equivalent sets of objects, say A and B. After the children had paired the objects and noted that each "a" had a "b" for a partner and each "b" had an "a" for a partner, the relationship, "there are as many a's as b's." was introduced. In a subsequent lesson, "there are more a's than b's," was defined by a situation in which it had been noted that when A was paired with B, there were some a's left which did not have a b for a partner. "Fewer than" was operationally defined in the analogous way.

Sample

The sample selected for this study was composed of the 51 children enrolled in Beech Haven Baptist Kindergarten and Green Acres Baptist Kindergarten in Athens, Georgia. Nine subjects were eliminated from the study. The final sample consisted

of 21 boys and 21 girls with ages in the range of 65 months to 75 months. The subjects may be classified as children from white, middle socio-economic class families.

Procedures

Because it was not assumed that Ss were familiar with the relations "as many as," "more than," and "fewer than," learning activities, intended to operationally define the relational terminology were provided. The seven 20-30 minute learning sessions took place during a two week period in April 1970. One of the experimenters instructed the children in one school while the regular teacher instructed the children in the other school. Approximately 13 children were in each instructional group.

After the instructional sequence was completed, the 18 item RT and CMRT was administered on an individual basis in a different, random sequence for each child. Two testers gave nine items each in an untimed interview which took approximately 15 to 20 minutes. Separate rooms were used by the examiners. Each testing session was begun with three warm-up items, one on each relation, which were intended to refamilarize Ss with matching sets of objects and establishing the relation between them. The examiner corrected any incorrect responses made to a question on a warm-up item.

A child was said to meet the criterion on RT if he established the relation on four of the six items on each relation. There were 16 Ss who failed to reach criterion on at least one relation. These children repeated selected activities from the learning sequence in three additional sessions. Upon repeating the RT and CMRT for deficient relations, seven children again failed to attain the criterion level on relational terminology; these were eliminated from further study.

The results of CMRT were used as a basis for classifying subjects as high

or low conservers of matching relations. A child was classified as a high conserver provided he conserved the relation on four of the six items on each relation and as a low conserver otherwise. The probability of meeting this criterion by guessing is less than .003. All Ss except one had scores either above the criterion level or appreciably below the criterion level. The one exception, whose score was barely below the criterion level, and a child who was of legal age to be in grade one were eliminated from the data analyses. Thus there were 27 high conservers and 15 low conservers in the study. Prior to the transitivity test, nine high conservers and five low conservers were randomly assigned to each relational category.

Three examiners* administered the TMRT in separate rooms which did not include a regular classroom. Each individual interview was preceded by a review of relational terminology in a concrete situation. Each examiner queried a child on two randomly chosen items for each of the stimulus conditions. These six items were administered in a randomly ordered sequence on three consecutive school days.

The number of items correct on each stimulus subtest was considered as response data for the analysis of variance. In order to study the relationship between a child's ability to conserve a relation and the development of transitivity of the relation it was necessary to set a criterion for transitivity performance. This criterion was defined by the following: a child uses the transitive property of a matching relation under a given stimulus condition provided he correctly responds to each question in four of the six items for that condition. The probability of achieving this criterion by guessing is less than .003.

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*The experimenters are indebted to Mr. James R. Devane, graduate student, and Mrs. Ronald S. McCuiston, wife of a graduate student, both of the University of Georgia, for their assistance as examiners.

Data Analyses

An internal consistency reliability for the conservation test was calculated using the Kuder-Richardson formula 20. Since measures on a set of TMRT items were available for only 14 subjects, a reliability study of these tests was not considered to be feasible.

A 2x3x3 design with repeated measures on the third factor was used to detect differences in the means or interactions among the factors of conservation, relation, and stimulus condition. In Table 1, L and H denote the low and high levels of conservation. AMA, MT, and FT denote the relations "as many as,"

Insert Table 1 about here

"more than," and "fewer than." The repeated measures occurred on the three stimulus conditions as indicated by the six groups Gi, i=1,2,...,6. All factors were considered fixed. The statistical analysis for the ANOVA is given by Winer (1962). Since the number of subjects in each of G1, G2 and G3 was five and in G4, G5, and G6 was nine, a least squares procedure was used to estimate sums of squares.

A Chi-square test was used to test the 2x2 contingency tables for independence between conservation and transitivity. Contingency tables for each relation and each stimulus condition were constructed and tested.

Results of the Study

The KR-20 reliability for the conservation test was .94. This high internal consistency coefficient indicates good homogenity of the test items.

The ANOVA, given in Table 2, revealed significant interaction between conservation level and relations. The means for this interaction are given to

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Insert Table 2 about here

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the nearest percent in Table 3. The interaction profile is presented in Figure 3. A Newman-Kuels test was performed on the simple effects of the CxR interaction to determine where the variation occurred. Since cell sizes were unequal, the harmonic mean of the frequencies was used to calculate the standard error of a mean. Within the high conservation level the mean of 77% for the relation AMA differs from the mean of 51% for the relation MT ($p<.10$) as well as from the mean of 46% for the relation FT ($p<.10$). The only relation on which high and low conservers differed significantly ($p<.05$) was the equivalence relation AMA.

Insert Table 3 about here

Insert Figure 3 about here

The ANOVA of Table 2 also showed that stimulus condition was a significant variable ($p<.05$). Since repeated measures occurred on this variable, a conservative test was used (Winer, 1962). The means (as percents) are 59, 51, and 47 for the neutral stimulus, screened stimulus and perceptual conflict, respectively. The Newman-Kuels test revealed that the mean for the neutral condition differed from the mean for the screening condition ($p<.05$) and from the mean for the conflict condition ($p<.01$).

Table 4 contains the frequency of Ss scoring above the criterion (high) and below the criterion (low) on conservation and transitivity for each stimulus condition. Since each S was assigned to a relation for the measurment on

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Insert Table 4 about here

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transitivity, the conservation level was considered only for the assigned relation.

There were Ss who were classified at the low level of conservation in the ANOVA, but who met the criterion on the assigned relation of Table 4. The observations on all relations were summed to obtain the frequencies of Table 4. The non-significant Chi-Square statistic, which was computed from the cell frequencies of the 2x2 table for the neutral stimulus condition, was 2.47 with 1 df. Significant Chi-Squares ($p < .05$) of 6.40 and 5.53 (each with 1 df) were obtained for the screened and conflicting stimuli, respectively.

The frequencies of Table 5 were obtained by summing the observations for the

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Insert Table 5 about here

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three stimulus conditions in each relational category. Since there are repeated measures for each subject on the stimulus factor, the 2x2 tabular frequencies for each relation are correlated. The Chi-Square of 21.57 with 1 df for the AMA table is significant ($p < .01$). The remaining Chi-Squares of 1.22 and 1.25 (1df) for the relations MT and FT are non-significant.

Discussion

A surprising result of the study is that Ss in the high category of conservation did not perform significantly better on transitivity than Ss in the low category of conservation for each relation. The significant interaction of conservation level and relational category and the results of the ensuing Newman-Kuel tests indicate that high level conservers perform better on transitivity of the equivalence relation "as many as," than their peers in the low conserver group. However, there is no difference in the performance of low and high conservers on transitivity of the order relations "more than" and "fewer than." The results further indicate that differential performance on transitivity between equivalence and order relational groups occurred only

within the high conservation level. Low conservation category Ss performed at about the same level in all relational categories. These conclusions from the ANOVA are consistent with the results of the Chi-Square tests for relational categories. When a criterion was set for conservation and transitivity performance and Chi-Square tests performed on the 2x2 contingency tables, the result was significant interaction between attainment of conservation and transitivity of the equivalence relation but independence between attainment of conservation and transitivity of the order relations.

The ANOVA revealed that stimulus condition is a significant factor. However the expected interaction of stimulus condition with conservation level did not occur. The Newman-Keuls test revealed that performance was significantly higher on the neutral stimulus test than either of the screened or conflict stimuli, but no difference was detected in the latter two. Chi-Square tests on the frequencies of the 2x2 tables for stimulus conditions resulted in independence between attainment of conservation and transitivity of matching relations in the presence of a neutral stimulus. However, the significant Chi-Squares for the screened and conflict stimuli, with further examination of the observed frequencies (Table 4), support the logical hypothesis that attainment of transitivity is contingent upon attainment of conservation of matching relations, under these conditions.

A possible explanation of higher scores in the neutral stimulus situation, is that perceptual cues were obtained from the stimulus arrays. This would allow subjects to make a perceptual discrimination between the collections A and C without using transitivity. This could account for the lack of dependence of transitivity attainment on conservation attainment in this case.

A comparison was made between the present results and Smedslund's (1963a) data for conservation and transitivity of discontinuous quantity. Smedslund's results for Ss within the age range, 65 months to 75 months, of this study are

recorded in Table 6. A Chi-Square test on the 2x2 cell frequencies of Table 6

Insert Table 6 about here

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provides no evidence that attainment of conservation and transitivity of discontinuous quantity are related.

A comparison of Smedslund's data in Table 6 and the present data from Table 4, reveals that a higher percentage of the present Ss lie in the high-high cell than is the case with Smedslund's Ss. First, the present data reveal that 71% of all Ss were at the high level of conservation of their respective relations while Smedslund found 45% of his subjects to pass the conservation of discontinuous quantity test. Secondly, the present data classify roughly one-half of these at the high level of transitivity, and Smedslund found only one-third of those passing conservation also passing the transitivity of discontinuous quantity test. If Smedslund's data is representative of the trend to be expected, there is a possibility that the instruction in the present study accounts for the increased performance. Of course, the converse is possible with the explanation that Smedslund's subjects did not understand the terminology or instructions. Another possible explanation for the discrepancy is that Smedlund's subjects were given the premises for the transitive inference by the examiner. In the present study, the child has to ascertain the premises from pairing the objects. Even though Smedslund required the child to repeat the premises several times in the presence of the stimulus materials, there remains the possibility that the child did not assimilate the information.

The discrepancy between the present results and Smedslund's data could be due to the difference in scoring procedures. Smedslund required Ss to give a correct response and explanation to at least one of two items in order to pass either the conservation or transitivity test. In the present study Ss were required to pass four of six items on each test.

Smedslund (1963a) observed that his conservation and transitivity tests contained uncontrolled differences. One difference is that he is measuring conservation in the presence of equivalent sets and transitivity of an order relation. A second factor is that the transitivity stimulus involved merely a spatial displacement of the collections while in the conservation test there was also a change in configuration. In the present study there is a difference in stimulus between the conservation and transitivity tests except in the conflict situation of the transitivity items. All items of the conservation test involved a conflict stimulus.

Smedslund (1963a) remarks that it seems that concrete reasoning is acquired in one restricted situation at a time. The present study supports this hypothesis by supplying evidence of differential performance among stimulus percepts. Further, it appears from these data that acquisition of transitivity of matching relations is relational specific.

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Table 1

Diagram of Design

Conservation Level	Relation	Stimulus Condition		
		Neutral	Screening	Conflict
H	AMA	G_1	G_1	G_1
	MT	G_2	G_2	G_2
	FT	G_3	G_3	G_3
L	AMA	G_4	G_4	G_4
	MT	G_5	G_5	G_5
	FT	G_6	G_6	G_6

Table 2

ANOVA For Conservation Levels and Relations

Source of Variation	df	MS	F
Conservation (C)	1	24.145	11.14**
Relation (R)	2	18.738	8.64**
C X R	2	14.215	6.56*
Subjects w. Groups	36	2.168	
Stimuli (S)	2	5.643	5.26†
C X S	2	1.466	1.37
R X S	4	.417	<1
C X R X S	4	1.562	1.46
C X Subjects w. Groups	72	1.072	

**p<.01

*p<.05

†p<.05, Conservative Test

Table 3

Means (As Percents): Conservation X Relation

Conservation Level	Relation			
	AMA	MT	FT	Total
High	77	51	46	58
Low	40	53	34	43
Total	64	52	42	52

Table 4

Frequencies by Stimuli Conditions:

Conservation X Transitivity

Conservation Level	Stimulus Condition					
	Neutral		Screened		Conflict	
	Level of Performance on Transitivity					
	High	Low	High	Low	High	Low
High	18	12	15	15	14	16
Low	4	8	1	11	1	11

Table 5

Frequency of Observations by Relation:

Conservation X Transitivity

Conservation Level	Relation					
	AMA		MT		FT	
	Level of Performance on Transitivity					
	High	Low	High	Low	High	Low
High	27	3	13	17	7	23
Low	2	10	3	9	1	11

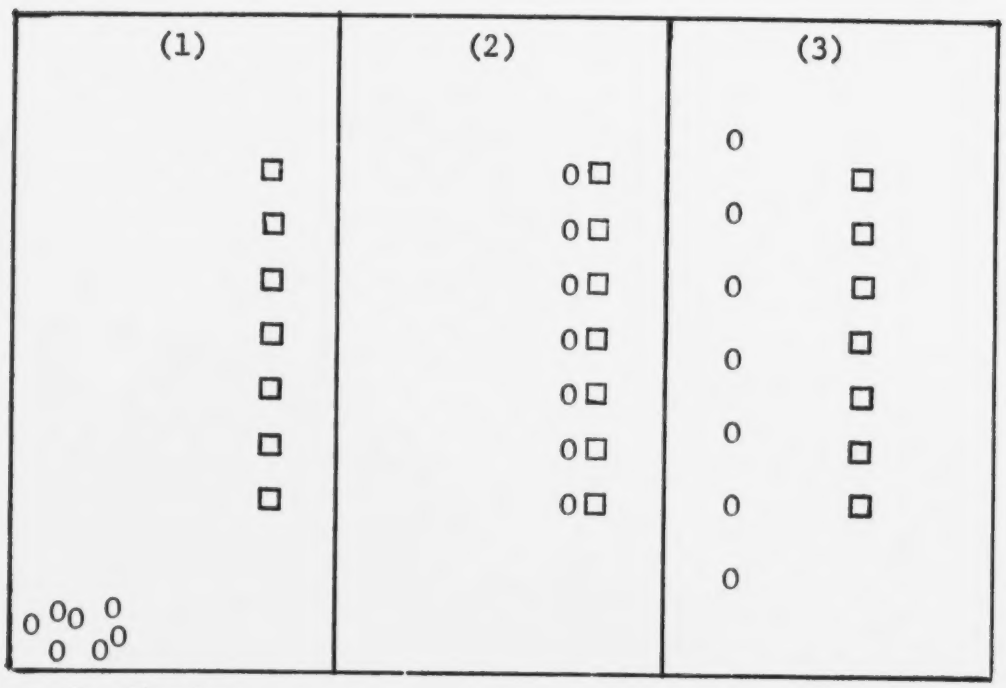
Table 6

Frequencies Observed by Smedslund

Discontinuous Quantity: Conservation X Transitivity

Conservation	Transitivity	
	Pass	Fail
Pass	5	10
Fail	3	15

Figure 1



O Checker

□ Tile

Figure 3

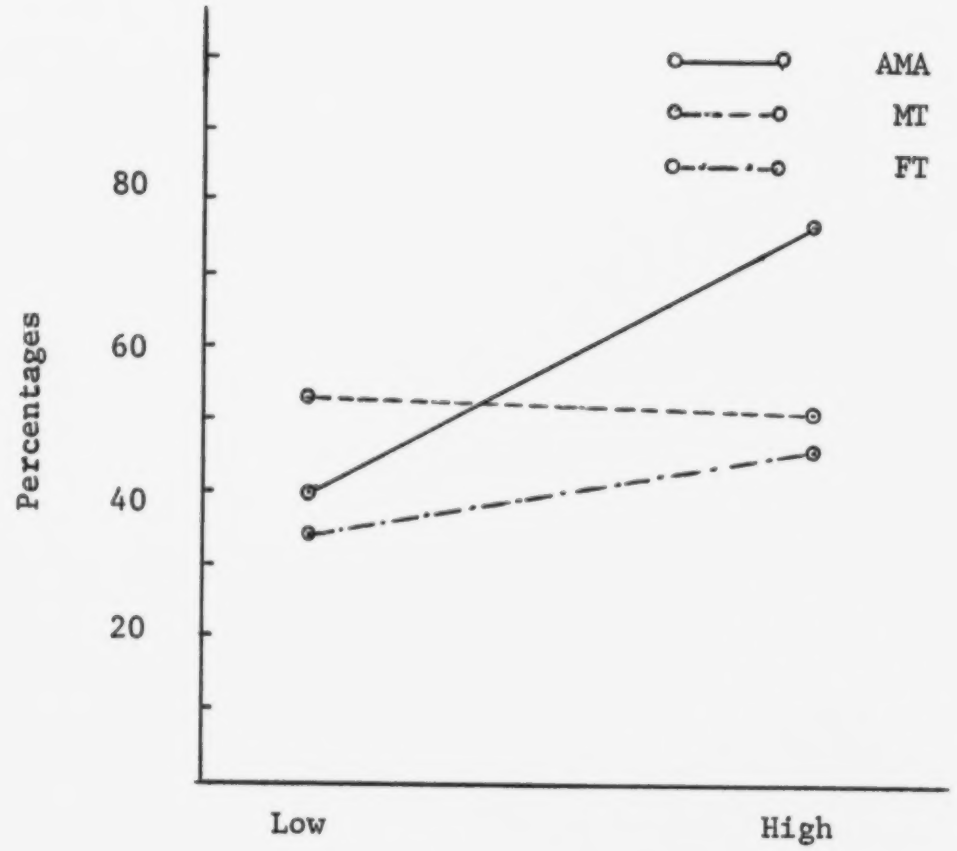
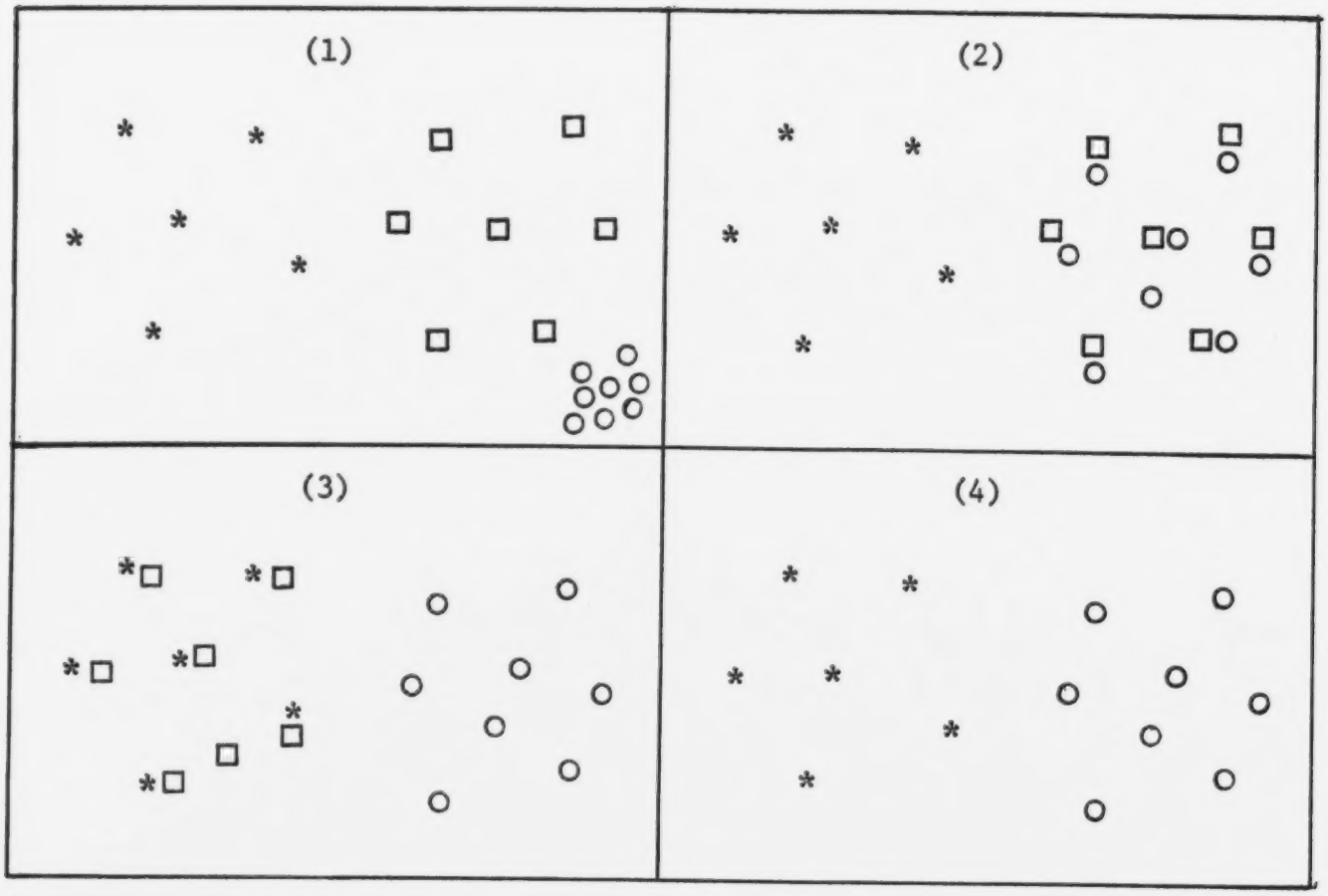

AMA
MT
FT
80
60
40
20
Percentages
Low
High
Level of Conservation

Figure 2



* Star

□ Tile

○ Checker